# Abstracts

# Lorenzo Brasco

Università degli Studi di Ferrara, Italy lorenzo.brasco@unife.it

#### Around Hardy's inequality for convex sets

We start by reviewing the classical Hardy inequality for convex sets. We then discuss the counterpart of Hardy's inequality for the case of fractional Sobolev-Slobodeckiĭ spaces, still in the case of open convex subsets of the Euclidean space. In particular, we determine the sharp constant in this inequality, by constructing explicit supersolutions based on the distance function. We also show that this method works only for the *mildly nonlocal* regime and it is bound to fail for the *strongly nonlocal* one. We conclude by presenting some open problems.

Some of the results presented are issued from papers in collaboration with Francesca Bianchi (Ferrara & Parma), Eleonora Cinti (Bologna), Firoj Sk (Oldenburg) and Anna Chiara Zagati (Ferrara & Parma).

# Elisa Davoli

TU Wien, Austria elisa.davoli@tuwien.ac.at

# Local asymptotics and optimal control for nonlocal Cahn-Hilliard systems

In this talk, we will consider a class of nonlocal Cahn-Hilliard equations both with Dirichlet and with Neumann boundary conditions for the chemical potential. After discussing local asymptotics in different settings, we will specify our analysis to a tumor growth model coupling a viscous Cahn-Hilliard equation describing the tumor proportion with a reaction-diffusion equation for the nutrient phase parameter. In particular, we will discuss first-order optimality conditions for suitable associated optimal control problems. This is based on a series of joint works with H. Ranetbauer, E. Rocca (Pavia), L. Scarpa (Milan), and L. Trussardi (Graz).

# Cristiana De Filippis

Universitá degli Studi di Parma, Italy cristiana.defilippis@unipr.it

#### Perturbations beyond Schauder

So-called Schauder estimates are a standard tool in the analysis of linear elliptic and parabolic PDE. They have been originally obtained by Hopf (1929, interior case), and by Schauder and Caccioppoli (1934, global estimates). The nonlinear case is a more recent achievement from the '80s (Giaquinta & Giusti, Ivert, Lieberman, Manfredi). All these classical results hold in the uniformly elliptic framework. I will present the solution to the longstanding problem, open since the '70s, of proving estimates of such kind in the nonuniformly elliptic setting. I will also cover the case of nondifferentiable functionals and provide a complete regularity theory for a new double phase model. From joint work with Giuseppe Mingione (University of Parma).

# Manuel Del Pino

University of Bath, UK m.delpino@bath.ac.uk

#### Gluing methods in the Water Wave Problem

In the classical Water Wave Problem, we construct new overhanging solitary waves by a procedure resembling desingularization of the gluing of constant mean curvature surfaces by tiny catenoidal necks. The solutions here predicted have long been numerically detected. This is joint work with Juan Davila, Monica Musso, and Miles Wheeler.

#### **Rupert Frank**

Ludwig-Maximilians-Universität München, Germany r.frank@lmu.de

Regularity for the Pauli zero mode system

For physical reasons one is interested in zero modes of the

For physical reasons one is interested in zero modes of the Pauli operator and, in particular, in the case where the magnetic field is minimal in some sense. The Euler-Lagrange equation for this optimization problem is a quasilinear system for a spinor field and a vector field and the equation is scaling-critical. We introduce the physical and mathematical background of this problem and present some results about existence, regularity and decay of solutions.

The talk is based on joint work with Michael Loss.

# Francesco Leonetti

Universitá degli Studi dell'Aquila, Italy

leonetti@univaq.it

#### Elliptic systems, double phase functionals and energy approximation

It is well known that solutions to elliptic systems may be unbounded. Nevertheless, for some special classes of systems, it can be proved that solutions are bounded. We mention a recent result of this kind and we discuss some examples suggested by double phase functionals. This leads us to the problem of energy approximation of functions and its connection with Lavrentiev phenomenon.

# **Giuseppe Mingione**

Universitá degli Studi di Parma, Italy

giuseppe.mingione@unipr.it

#### Schauder estimates at nearly linear growth

Schauder estimates are a classical tool in linear and nonlinear elliptic and parabolic PDEs. They describe how regularity of coefficients reflects in regularity of solutions. They basically have a perturbative nature. This means that they can be obtained by perturbing the estimates available for problems without coefficients. This paradigm works as long as one deals with uniformly elliptic equations. The nonuniformly elliptic case is a different story and Schauder's theory turns out to be not perturbative any longer, as shown by counterexamples. In my talk, I will present a method allowing to bypass the perburbative schemes and leading to Schauder estimates in the nonuniformly elliptic regime. For this I will concentrate on the case of nonuniformly elliptic functionals with nearly linear growth, also covering a borderline case of so-called double phase energies. From recent, joint work with Cristiana De Filippis (Parma).

# **Guofang Wang**

Albert-Ludwigs-Universität Freiburg, Germany

guofang.wang@math.uni-freiburg.de

### Capillary hypersurfaces

I will talk about our new results on capillary hypersurfaces: Alexandrov-Fenchel inequalities, Heintze-Karcher inequality, curvature type flow and related problems.

The talk is based on the joint work with Chao Xia (Xiamen University) and other colleagues.